MGSC 1205

Quantitative Methods I

Slides Six – Formulation & Sensitivity Analysis I

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How can we handle changes?

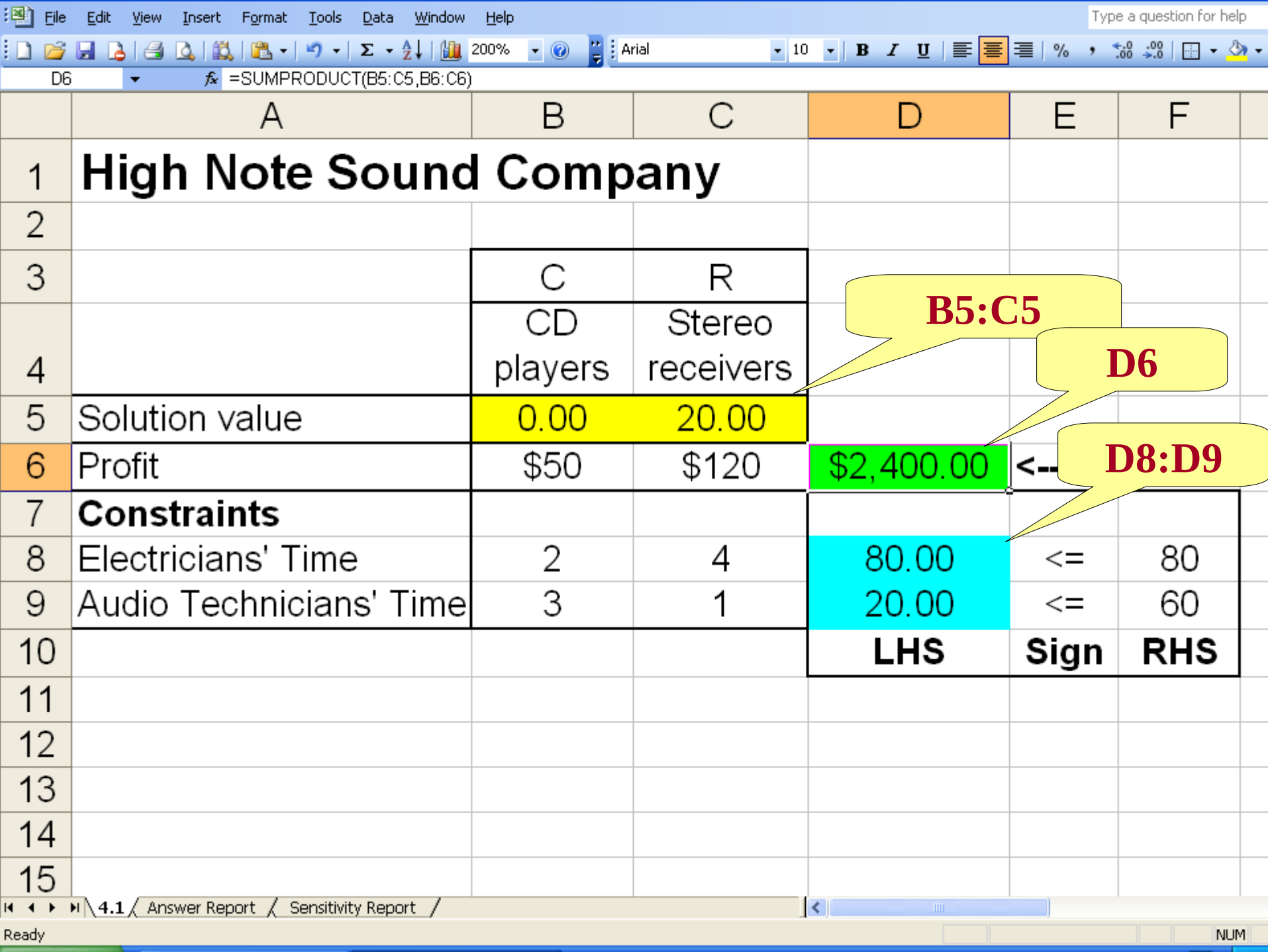
- We have solved *LP* problems under **deterministic** assumptions.
  - find an optimum solution given certain constant parameters (costs, price, time, etc)
- How well do we know these parameters?
  - Usually not very accurately – rough estimates
  - Conditions in most world situations are dynamic & changing
    - ✓ prices of raw materials change
    - ✓ product supply changes
    - ✓ new machinery is bought to replace old
    - ✓ employee turnover occurs …

Sensitivity Analysis

- Post-optimality analysis: examining changes after the optimal solution has been reached.
  - input data are varied to assess optimal solution sensitivity.
- **Basic Question:** How does our solution change as the input parameters change?
  - How much does the objective function change?
  - How much do the optimal values of the decision variables change?
  - **Do our results remain valid** (If the parameters change...)?

Example: High Note Sound Company

- The company Manufactures quality CD players and stereo receivers.
- Each CD player sold results in $50 profit, while each receiver yields $120 profit.
- Each product requires skilled craftsmanship.
  - **Each CD player requires:** 2 hours electrician's time and 3 hours technician's time
  - **Each receiver requires:** 4 hours electrician's time and 1 hour technician's time
- **Hours available:** 80 for electrician's time, 60 for technician's time
- **Objective:** maximize profit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High Note Sound Company
C
R
CD players
Stereo receivers
Solution value
0.00
20.00
Profit
$50
$120
$2,400.00
<--
Constraints
Electricians' Time
2
4
80.00
<=
80
Audio Technicians' Time
3
1
20.00
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60
LHS
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B5:C5
D6
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4.1
Answer Report
Sensitivity Report
Ready
NUM

Answer Report

Optimal Objective function value

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- This table gives us information obtained from the objective function.
  - ✓ which cell the objective function is located in,
  - ✓ its initial value before solver was initiated,
  - ✓ the value obtained by plugging in the values of the decision variables from the optimal corner point.

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.0

Optimal solution values

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- This table gives us information on the decision variables.
  - ✓ which cells the decision variables are located in
  - ✓ their initial values before solver was initiated
  - ✓ their values corresponding to the optimal solution

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- This table gives us information the constrains.
  - ✓ Cell value: how much of the given resource is used up in obtaining the optimal solution
  - ✓ Formula: the constraint equation in cell notation

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	Solution value CD players	0.00	0.00
$C$5	Solution value Stereo receivers	0.00	20.00

This column indicates whether a constraint is exactly satisfied (*LHS*=*RHS*)

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- **Binding** means the constrain is exactly satisfied, and *LHS* = *RHS*.
  - All the available resource is fully used in the solution
  - Nonbinding means that some of the resource has not been fully used up in the final solution

Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$6	Profit	$0.00	$2,400.00

Adjustable Cells

Cell	Name	Original Value	Final V
$B$5	Solution value CD players	0.00	
$C$5	Solution value Stereo receivers	0.00	

This column indicates the amount of unused resource

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$8	Electricians' Time	80.00	$D$8<=$F$8	Binding	0.00
$D$9	Audio Technicians' Time	20.00	$D$9<=$F$9	Not Binding	40.00

- **Slack** is the difference between the *RHS* and the *LHS* of a ≤ constrain
  - Binding constrain: slack =0.
  - A nonbinding constrain is when the *slack* > 0.

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Electricians' Time	80.00	30.00	80.00	160.00	80.00
$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

- We are analyzing only one change at a time
- This table presents information regarding the impact of changes to the *OFC*s (i.e., the unit profits of $50 & $120)
  - ✓ **Allowable Increases & Allowable Decreases**: they are the range of values for which we can change the *OFC*s, and still have current **Corner Point** remain as **Optimal Solution**
  - **This is the whole point of doing the analysis!**

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Electricians' Time	80.00	30.00	80.00	160.00	80.00
$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

- ✓ It is allowed to increase the *OFC* value by up to $10 and still have no change in optimal solution.
- ✓ Excel's notation for ∞: the price of CD players can be dropped by ∞ and still has no change in optimal solution (we still won't make any).

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
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$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

At what point would we want to start making CD player?

✓ Back to **Reduced Cost**.

- Losing $10 for each CD player that we choose to make with its current *OFC*.
- If we can somehow raise the OFC by $10 per unit, making CD players stops losing money; If we raise it more than $10, then producing CD players becomes profitable
- **This is exactly the Allowable Increase for CD player!**

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	Solution value CD players	0.00	-10.00	50.00	10.00	1E+30
$C$5	Solution value Stereo receivers	20.00	0.00	120.00	1E+30	20.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$8	Electricians' Time	80.00	30.00	80.00	160.00	80.00
$D$9	Audio Technicians' Time	20.00	0.00	60.00	1E+30	40.00

- ✓ **Allowable Increases** & **Allowable Decreases**: they are the range of values for which we can change the *OFC*s, and still have current **Corner Point** remain as **Optimal Solution**

Sensitivity Analysis

- We have solved LP problems under deterministic assumptions. i.e., finding an optimum solution given certain constant parameters (costs, price, time, etc.)
- Conditions in most world situations are dynamic & changing
  - prices of raw materials change, product supply changes, new machinery is bought to replace old, employee turnover occurs …
- Post-optimality analysis: examining changes after the optimal solution has been reached.
- **Basic Question: How does our solution change as the input parameters change?**
  - **Do our results remain valid (If the parameters change...)?**